RODOLFO GUIDOBONO **CURSO TEORÍA CUÁNTICA DE CAMPOS** **JAVIER GARCÍA**
EJERCICIO CAPÍTULO 63

Verificar que los campos E y B son invariantes bajo transformaciones Gauge.

Considerar

1) $A^0 = V = A_0;\ A^1 = A_x = -A_1;\ A^2 = A_y = -A_2;\ A^3 = A_z = -A_3$

2) $\partial^0 = \partial_0\ ;\ \partial^a = -\partial_a\ para\ a = 1, 2, 3$

3) $$\boldsymbol{E} = -\nabla \cdot \mathrm{V} - \frac{\partial\,\mathbf{A}}{\partial\,\mathrm{t}} = \begin{pmatrix} -\frac{\partial \mathrm{V}}{\partial \mathrm{x}} - \frac{\partial A_x}{\partial \mathrm{t}} \\ -\frac{\partial \mathrm{V}}{\partial \mathrm{y}} - \frac{\partial A_y}{\partial \mathrm{t}} \\ -\frac{\partial \mathrm{V}}{\partial \mathrm{z}} - \frac{\partial A_z}{\partial \mathrm{t}} \end{pmatrix}$$

4) $$\boldsymbol{B} = \nabla \times \mathbf{A} = \begin{pmatrix} \frac{\partial A_z}{\partial \mathrm{y}} - \frac{\partial A_y}{\partial \mathrm{z}} \\ -\frac{\partial A_z}{\partial \mathrm{x}} + \frac{\partial A_x}{\partial \mathrm{z}} \\ \frac{\partial A_y}{\partial \mathrm{x}} - \frac{\partial A_x}{\partial \mathrm{y}} \end{pmatrix}$$

5) $A'_\mu = A_\mu - \frac{1}{g}\partial_\mu \theta \equiv A_\mu - \partial_\mu f_{(x)}$

$$A'_0 = A_0 - \partial_0 f$$
$$A'_1 = A_1 - \partial_1 f$$
$$A'_2 = A_2 - \partial_2 f$$
$$A'_3 = A_3 - \partial_3 f$$

Campo Eléctrico

$$E_x = -\partial_x V - \partial_0 A_x = -\partial_1 A^0 - \partial_0 A^1 = -\partial_1 A_0 + \partial_0 A_1 = E^1 = -E_1$$

$$E_1 = \partial_1 A_0 - \partial_0 A_1$$

$$E'_1 = \partial_1 A'_0 - \partial_0 A'_1$$

$$E'_1 = \partial_1(A_0 - \partial_0 f) - \partial_0(A_1 - \partial_1 f) = \partial_1 A_0 - \partial_1 \partial_0 f - \partial_0 A_1 + \partial_0 \partial_1 f = \partial_1 A_0 - \partial_0 A_1$$

$$\boldsymbol{E_1 = E'_1}$$

$$E_y = -\partial_y V - \partial_0 A_y = -\partial_2 A^0 - \partial_0 A^2 = -\partial_2 A_0 + \partial_0 A_2 = E^2 = -E_2$$

$$E_2 = \partial_2 A_0 - \partial_0 A_2$$

$$E'_2 = \partial_2 A'_0 - \partial_0 A'_2$$

$$E'_2 = \partial_2(A_0 - \partial_0 f) - \partial_0(A_2 - \partial_2 f) = \partial_2 A_0 - \partial_2 \partial_0 f - \partial_0 A_2 + \partial_0 \partial_2 f = \partial_2 A_0 - \partial_0 A_2$$

$$\boldsymbol{E_2 = E'_2}$$

$$E_z = -\partial_z V - \partial_0 A_z = -\partial_3 A^0 - \partial_0 A^3 = -\partial_3 A_0 + \partial_0 A_3 = E^3 = -E_3$$

$$E_3 = \partial_3 A_0 - \partial_0 A_3$$

$$E'_3 = \partial_3 A'_0 - \partial_0 A'_3$$

$$E'_3 = \partial_3(A_0 - \partial_0 f) - \partial_0(A_3 - \partial_3 f) = \partial_3 A_0 - \partial_3 \partial_0 f - \partial_0 A_3 + \partial_0 \partial_3 f = \partial_3 A_0 - \partial_0 A_3$$

$$\boldsymbol{E_3 = E'_3}$$

Campo Magnético

$$B_x = \frac{\partial A_z}{\partial \mathrm{y}} - \frac{\partial A_y}{\partial \mathrm{z}} = \partial_2 A^3 - \partial_3 A^2 = -\partial_2 A_3 + \partial_3 A_2 = B^1 = -B_1$$

$$B_1 = \partial_2 A_3 - \partial_3 A_2$$

$$B'_1 = \partial_2 A'_3 - \partial_3 A'_2$$

$$B'_1 = \partial_2(A_3 - \partial_3 f) - \partial_3(A_2 - \partial_2 f) = \partial_2 A_3 - \partial_2 \partial_3 f - \partial_3 A_2 + \partial_3 \partial_2 f = \partial_2 A_3 - \partial_3 A_2$$

$$\boldsymbol{B_1 = B'_1}$$

$$B_y = -\frac{\partial A_z}{\partial \mathrm{x}} + \frac{\partial A_x}{\partial \mathrm{z}} = \partial_1 A^3 - \partial_3 A^1 = -\partial_1 A_3 + \partial_3 A_1 = B^1 = -B_1$$

$$B_2 = \partial_1 A_3 - \partial_3 A_1$$

$$B'_2 = \partial_1 A'_3 - \partial_3 A'_1$$

$$B'_2 = \partial_1(A_3 - \partial_3 f) - \partial_3(A_1 - \partial_1 f) = \partial_1 A_3 - \partial_1 \partial_3 f - \partial_3 A_1 + \partial_3 \partial_1 f = \partial_1 A_3 - \partial_3 A_1$$

$$\boldsymbol{B_2 = B'_2}$$

$$B_z = \frac{\partial A_y}{\partial \mathrm{x}} - \frac{\partial A_x}{\partial \mathrm{y}} = \partial_1 A^2 - \partial_2 A^1 = -\partial_1 A_2 + \partial_2 A_1 = B^1 = -B_1$$

$$B_3 = \partial_1 A_2 - \partial_2 A_1$$

$$B'_3 = \partial_1 A'_2 - \partial_2 A'_1$$

$$B'_3 = \partial_1(A_2 - \partial_2 f) - \partial_2(A_1 - \partial_1 f) = \partial_1 A_2 - \partial_1 \partial_2 f - \partial_2 A_1 + \partial_2 \partial_1 f = \partial_1 A_2 - \partial_2 A_1$$

$$\boldsymbol{B_3 = B'_3}$$